

Review of Computational Approach to Finite Element Analysis of Slab

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Abstract – Slabs are one of the most widely used structural elements. The function of slabs is to resist loads normal to their plane. In many structures, in addition to support transverse load, the slab also forms an integral portion of the structural frame to resist lateral load. The paper presents review of computational approach to finite element analysis of slabs. The finite element method is chosen as this is more powerful and versatile compared to other numerical methods.

Index Terms- Slabs, Plates, Boundary conditions, Finite Element Method, Displacement function, Differential equation, element wise.

1. INTRODUCTION

Slabs are most widely used structural elements of modern structural complexes and the reinforced concrete slab is the most useful discovery for supporting lateral loads in buildings. Slabs may be viewed as moderately thick plates that transmit load to the supporting walls and beams and sometimes directly to the columns by flexure, shear and torsion. It is because of this complex behaviour it is difficult to decide whether the slab is a structural element or structural system in itself. Slabs are viewed in this paper as a structural element. The greatest volume of concrete that goes into a structure is in the form of slabs, floors and footings. Since slabs have a relatively large surface area compared with their volume, they are affected by temperature and shrinkage. Slabs may be visualized as intersecting, closely spaced, grid-beams and hence they are seen to be highly indeterminate. This high degree of indeterminacy is directly helpful to designer, since multiple load-flow paths are available and approximations in analysis and design are compensated by heavy cracking and large deflections, without significantly affecting the load carrying capacity. Slabs being highly indeterminate, are difficult to analyze by elastic theo-

ries. More recently, finite difference and finite element methods have been introduced and this is extremely useful. Methods have also been innovated to find the collapse loads of various types of slabs through the yield line theory and strip methods. In addition to supporting lateral loads (perpendicular to the horizontal plane), slabs act as deep horizontal girders to resist wind and earthquake forces that act on a multi-storied frame. Their action as girder diaphragms of great stiffness is important in restricting the lateral deformations of a multi-storied frame. However, it must be remembered that the very large volume and hence the mass of these slabs are sources of enormous lateral forces due to earthquake induced accelerations.

2. PLATE THEORY

In continuum mechanics, plate theories are mathematical descriptions of the mechanics of flat plates that draws on the theory of beams. Plates are defined as plane structural elements with a small thickness compared to the planar dimensions[1]. Plates are very important structural elements. They are mainly used as slabs in buildings and bridge decks. They are structural elements that are bound

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by two lateral surfaces. The dimensions of the lateral surfaces are very large compared to the thickness of the plate. A plate may be thought of as the two-dimensional equivalent of a beam. Plates are also generally subject to loads normal to their plane.

2.1 Thin Plates

Classical Plate Theory

The Kirchhoff-Love Theory of Plates:

The Kirchhoff-Love theory is an extension of Euler-Bernoulli beam theory to thin plates. The theory was developed in 1888 by Love[2] The small deflection theory of plates attributed to Kirchhoff is based on the following assumptions[3]:

- The xy plane coincides with the middle plane of the plate in the undeformed geometry.
- The lateral dimension of the plate is at least 10 times its thickness.
- The vertical displacement of any point of the plate can be taken equal to that of the point (below or above it) in the middle plane.
- A vertical element of the plate before bending remains perpendicular to the middle surface of the plate after bending.
- Strains are small: deflections are less than the order of $(1/100)$ of the span length.
- The strain of the middle surface is zero or negligible.

Considering the plate element shown in Fig (1), the in-plane displacements u and v , respectively in the directions x and y , can be expressed as:

$$u = -z\partial w/\partial x \quad (1)$$

$$v = -z\partial w/\partial y \quad (2)$$

where w represents the vertical displacement of the plate mid-plane.

Because of the assumption, 'a vertical element of the plate before bending remains perpendicular to the middle surface of the plate after bending,' the transverse shear deformation is negligible. The in-plane strains can therefore be written in terms of the displacements as

$$\begin{aligned} \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} &= \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{Bmatrix} \\ &= \begin{Bmatrix} -z^2 \frac{\partial^2 w}{\partial x^2} \\ -z^2 \frac{\partial^2 w}{\partial y^2} \\ -2z^2 \frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} = -z^2 \begin{Bmatrix} \chi_x \\ \chi_y \\ \chi_{xy} \end{Bmatrix} \end{aligned} \quad (3)$$

The vector $\chi = [\chi_x \chi_y \chi_{xy}]^T$ is called the vector of curvature or generalized strain.

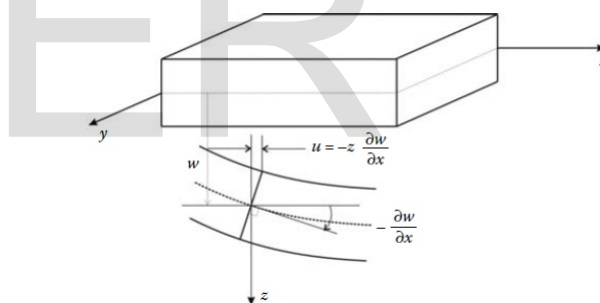


Figure 1: Deformed configuration of a thin plate in bending

Internal stresses in plates produce bending moments and shear forces as illustrated in Fig (2) and Fig (3). The moments and shear forces are the resultants of the stresses and are defined as acting per unit length of plate. These internal actions are defined as

$$M_{xx} = \int_{-h/2}^{h/2} \sigma_{xx} z dz \quad (4)$$

$$M_{yy} = \int_{-h/2}^{h/2} \sigma_{yy} z dz \quad (5)$$

$$M_{xy} = \int_{-h/2}^{h/2} \tau_{xy} z dz \quad (6)$$

$$Q_{xx} = \int_{-h/2}^{h/2} \sigma_{xx} z dz \quad (7)$$

$$Q_{yy} = \int_{-h/2}^{h/2} \sigma_{yy} z dz \quad (8)$$

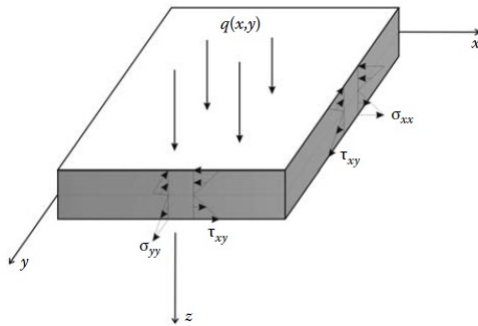


Figure 2: Internal stresses in a thin plate.

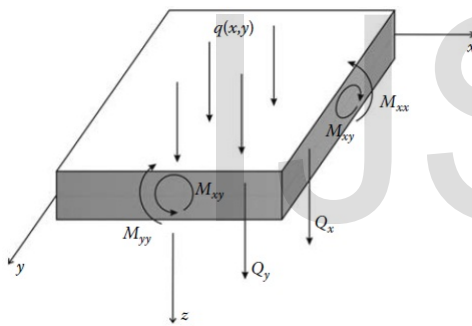


Figure 3: Moments and shear forces due to internal stresses in a thin plate.

In general, the force and moment intensities vary with the coordinates x and y of the middle plane. Assuming a state of plane stress conditions for plate bending,

$$\sigma = [D]\epsilon \quad (9)$$

with $[D]$ given as

$$[D] = \frac{E}{1-\nu^2} \begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{(1-\nu)}{2} \end{pmatrix} \quad (10)$$

and substituting for $\{\epsilon\}$ using Equation(3) yields the constitutive equation

$$\sigma = -z[D]\chi \quad (11)$$

Substituting for σ_{xx} , σ_{yy} , and τ_{xy} in Equation(4) and rearranging the results in a matrix notation yields

$$M = \frac{h^3}{12}[D]\chi \quad (12)$$

Consider the equilibrium of the free body of the differential plate element shown in Fig(4). Recalling that Q_x represents force per unit length along the edge dy and requiring force equilibrium in z direction results in

$$\begin{aligned} -Q_x dy - Q_y dx + (Q_x + \frac{\partial Q_x}{\partial x} dx) dy \\ + (Q_y + \frac{\partial Q_y}{\partial y} dy) dx + q(x,y) dx dy \end{aligned} \quad (13)$$

which upon dividing by $dx dy$ becomes

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q(x,y) = 0 \quad (14)$$

Moment equilibrium about the x -axis leads to

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_{yy}}{\partial y} = Q_y \quad (15)$$

Moment equilibrium about the y -axis leads to

$$\frac{\partial M_{xy}}{\partial y} + \frac{\partial M_{xx}}{\partial x} = Q_x \quad (16)$$

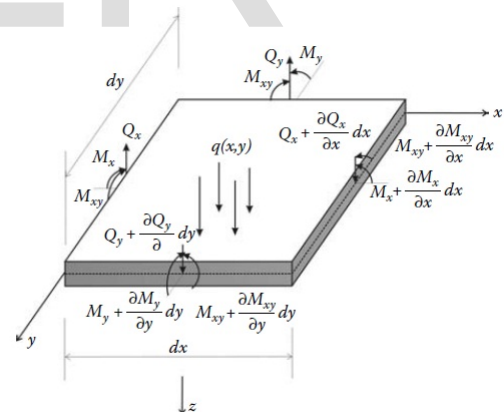


Figure 4: Free body diagram of a plate element.

Substituting (15) and (16) in (14) results in the governing equation

$$\frac{\partial^2 M_{xx}}{\partial x^2} + \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_{yy}}{\partial y^2} + q(x,y) = 0 \quad (17)$$

Since no relations regarding material behavior have entered Equation (17), it is valid for all types of materials.

2.2 Thick Plates

First-order shear plate theory

The Mindlin-Reissner theory of plates:

In the theory of thick plates, or theory of Raymond Mindlin[4] and Eric Reissner, the normal to the mid-surface remains straight but not necessarily perpendicular to the mid-surface. As explained previously, the Kirchhoff plate theory does not include shear deformations. This is an acceptable assumption for very thin plates, but it can lead to errors, which are not negligible in thick plates; most of reinforced concrete slabs are classified in this latter category. In thick plates, the assumption that a vertical element of the plate before bending remains perpendicular to the middle surface of the plate after bending is relaxed. Transverse normal may rotate without remaining normal to the mid-plane. A line originally normal to the middle plane will develop rotation components θ_x relative to the middle plane after deformation as shown in Fig(5). A similar definition holds for θ_y . Hence, the displacement field becomes

$$u = z\theta_x \tag{18}$$

$$v = z\theta_y \tag{19}$$

$$w = w(x, y) \tag{20}$$

The strains associated with these displacements are given as

$$\begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{pmatrix} = \begin{pmatrix} z \frac{\partial \theta_x}{\partial x} \\ z \frac{\partial \theta_y}{\partial y} \\ z \left(\frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} \right) \\ z \left(\theta_y - \frac{\partial w}{\partial y} \right) \\ z \left(\theta_x - \frac{\partial w}{\partial x} \right) \end{pmatrix} \tag{21}$$

These equations are the main equations of the Mindlin plate theory. The theory accounts for transverse shear deformations and is applicable for moderately thick plates. Unlike in thin plate theory, it is important to notice that the transverse displacement $w(x, y)$ and slopes θ_x, θ_y are independent. Notice also

that the thick plate theory reduces to thin plate theory if

$$\theta_x = -\frac{\partial w}{\partial x}$$

and

$$\theta_y = -\frac{\partial w}{\partial y}$$

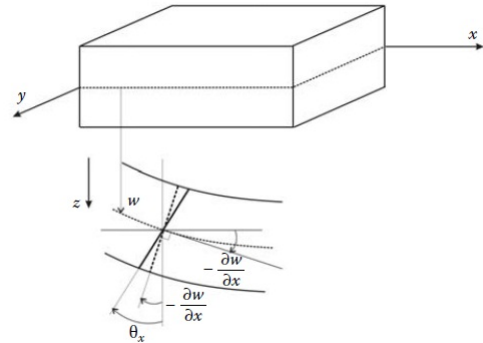


Figure 5: Deformed configuration of a thick plate in bending.

2.3 Boundary Conditions

Given a rectangular plate with dimensions $a \times b \times h$ as shown in Fig(6). The governing equation of the bending behaviour of a thin plate is described by a fourth-order differential equation. Hence, two boundary conditions have to be specified on each edge[5].

2.3.1 Simply Supported Edge:

If the edge $x = a$ is simply supported, the deflection $(w)_{x=a}$ along this edge must be zero. At the same time, the edge can rotate freely with respect to the support, that is, there is no bending moment M_{xx} along this edge: $a \times b \times h$

$$(w)_{x=a} = 0$$

and

$$(M_{xx})_{x=a} = -D_r \left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right) \tag{22}$$

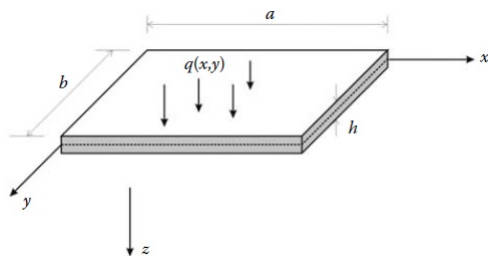


Figure 6: Plate boundary conditions.

The condition $(w)_{x=a} = 0$ along the edge $x = a$ means also that $\frac{\partial w}{\partial y} = \frac{\partial^2 w}{\partial^2 y} = 0$ along that edge. The boundary conditions for a simply supported edge may also be written as

$$(w)_{x=a} = 0 \quad \text{and} \quad \frac{\partial^2 w}{\partial x^2} = 0 \quad (23)$$

The first boundary condition in (23) is a kinematic boundary condition and the second one is a dynamic or natural boundary condition.

2.3.2 Built-in or Clamped Edge:

If the edge $x = a$ is built-in or clamped, along this edge the deflection and the slope of the middle plane must be zero; that is,

$$(w)_{x=a} = 0 \quad \text{and} \quad \frac{\partial w}{\partial x} = 0 \quad (24)$$

These boundary conditions are both kinematic and need to be imposed.

2.3.3 Free Edge:

If the edge $x = a$ is entirely free, it is natural to assume that along this edge there are no bending and twisting moments, and also no shear force; that is

$$(M_{xx})_{x=a} = (M_{xy})_{x=a} = (Q_{xz})_{x=a} = 0 \quad (25)$$

Within the thin plate theory, these three conditions are combined into two conditions, namely

$$(M_{xx})_{x=a} = 0$$

and

$$(Q_{xz} + \frac{M_{xy}}{\partial y})_{x=a} = 0 \quad (26)$$

The term $Q_{xz} + \frac{M_{xy}}{\partial y}$ is called the 'effective shear force' or the 'Kirchhoff shear force.' The boundary conditions at a free edge are all natural and do not need to be imposed.

3. HISTORICAL BACKGROUND OF FINITE ELEMENT METHOD

The birth of variational calculus and the principle of virtual work goes back to the 17th and 18th century, and the first draft of a discrete variational method with "element wise" triangular shape functions was given by Leibniz (1697). First analytical studies were made by Schellbach (1851) and then, already with numerical results, by Rayleigh (1877). The mathematician Ritz (1909) marks the first discrete (direct) variational method for the linear elastic Kirchhoff plate, and the engineer Galerkin (1915) published his seminar article on FEM for linear elastic continua, postulating the orthogonality of the residua of equilibrium with respect to the test functions, but both, Ritz and Galerkin, used test and trial functions within the whole domain as supports. In 1921, Westergaard and Slater[6] correlated the results of experiments in slabs with the analytical theories at the time, so that building regulations for the slabs can be made. In 1943 Courant [7] made an effort to use piecewise continuous functions defined over triangular domain. After that it took nearly a decade to use this distribution idea. In fifties renewed interest in this field was shown by Polya[8], Hersh[9] and Weinberger[10]. In 1953 Sutherland, Goodman and Newmark[11] obtained an approximate numerical solution for the case of slabs supported on elastic beams. The Ritz energy method is used to obtain solutions for an interior panel of a plate or slab which is continuous over a rectangular grid of flexible beams supported by columns at their intersections. Argyris and Kelsey[12] introduced the concept of applying energy principles to the formation of structural analysis problems in 1960. In the same year Clough[13] introduced the

word 'Finite Element Method'. In sixties convergence aspect of the finite element method was pursued more rigorously. One such study by Melesh[14] led to the formulation of the finite element method based on the principles of minimum potential energy. Soon after that de Veubeke[15] introduced equilibrium elements based on the principles of minimum potential energy. Pion[16] introduced the concept of hybrid element using the dual principle of minimum potential energy and minimum complementary energy. In Late 1960's and 1970's, considerable progress was made in the field of finite element analysis. The improvements in the speed and memory capacity of computers largely contributed to the progress and success of this method. In the field of solid mechanics from the initial attention focused on the elastic analysis of plane stress and plane strain problems, the method has been successfully extended to the cases of the analysis of three dimensional problems, stability and vibration problems, non-linear analysis. In 1972, Gamble[17] presented the results of a study of the influence of the stiffness of the supporting beams on the distribution of moments within typical interior panels of reinforced concrete floor slabs. The results are presented in terms of a beam stiffness parameter and the panel shape. It is shown that once the beam stiffness parameter exceeds 2 the moment distributions are relatively insensitive to further increases in beam stiffness. In 1973, Ramesh and Datta[18] developed a yield-line theory taking into account the compressive membrane action present in the slab-beam system having different degrees of edge restraint and different percentages of steel in the slab. The theory uses a rigid plastic strip approximation and takes into account the lateral bowing of the edge beam.

4. FINITE ELEMENT METHOD

4.1 Introduction

The Finite Element Method (FEM) is a numerical technique to find approximate solutions of partial differential equations. It was originated from the need of solving complex elasticity and structural analysis problems in Civil, Mechanical and Aerospace engi-

neering.

The finite element method[19] can be considered as a generalized displacement method for two and three dimensional continuum problems. It is necessary to discretize the continuum into a system with a finite number of unknowns so that the problems can be solved numerically. The finite element procedure can be divided into the following steps:

- Idealization of the continuous surface as an assembly of discrete elements.
- Selection of displacement models.
- Derivation of the element stiffness matrix.
- Assembly of element stiffness matrix into an overall structure stiffness matrix.
- Solution of the system of linear equations relating nodal point loads and unknown nodal displacements.
- Computation of internal stress resultants by use of the nodal point displacements already found.

4.2 Formulation of the Problem

4.2.1 Displacement Function:

In order to assure convergence to a valid result by mesh reinforcement, the following three sacred rules have emerged for the assumed displacement functions:

- The displacement must be continuous within the element and the displacements must be compatible between adjacent elements. For plane stress and plane strain elements, continuity of the displacement functions along is sufficient, whereas for bending elements, continuity of both the displacement and slope is needed.
- The displacement function must include the states of constant strain of the element. This seems to be the most sacred of all the rules, since eventually, by mesh

reduction, one is evitable going to reach small region where the strains are constant.

- The displacement function must allow the element to undergo rigid body motion without any internal strain. For plane stress and plate bending elements, it is easy to establish displacement functions satisfying all these three requirements.

$$\begin{aligned} u(x, y) &= a_1xy + a_2x + a_3y + a_4 \\ v(x, y) &= a_5xy + a_6x + a_7y + a_8 \\ w(x, y) &= a_9x^3y + a_{10}x^3 + a_{11}x^2y + a_{12}x^2 \\ &\quad + a_{13}xy^3 + a_{14}xy^2 + a_{15}xy + a_{16}x \\ &\quad + a_{17}y^3 + a_{18}y^2 + a_{19}y + a_{20} \end{aligned}$$

Alternatively, in matrix form we can write this symbolically as follows:

$$\{\bar{u}\} = [P]\{ai\} \quad (27)$$

Where $\{\bar{u}\}$ is vector of slab displacement and $[P]$ is matrix of displacement functions. Here the rectangular co-ordinate system is considered. The degree of freedom considered at each node (corner) of the element is u, v, w, wx and wy .

4.2.2 Element Stiffness Matrix:

To simplify the derivation of the element stiffness matrix, a more convenient form of nodal displacement parameters with five degrees of freedom per node is listed as follows:

$$\begin{aligned} [ui]^T &= \\ u1, v1, w1, w1x, w1y, u2, v2, w2, w2x, w2y, \\ u3, v3, w3, w3x, w3y, u4, v4, w4, w4x, w4y \end{aligned}$$

Where, $w_ix = (\delta_w/\delta_x)_i, w_iy = (\delta_w/\delta_y)_i$; $i = 1$ to 4 , stands for the node number of the node of an element.

Substituting the values of co-ordinates of four nodes in the three displacement function and two derivatives of w stated above, we get the nodal displacements of an element as follows:

$$\{ui\} = [H]\{ai\} \quad (28)$$

Where, $\{ui\}$ is vector of nodal displacement co-ordinates and $[H]$ is called transformation matrix. The strain displacement relationships used in the analysis of this of slab element may be expressed as:

$$\{e\} = [\delta]\{\bar{u}\} \quad (29)$$

Therefore substituting Equation(27) into Equation(29) we get the strain expressed in terms of displacement parameters as follows:

$$\begin{aligned} \{e\} &= [\delta]\{\bar{u}\} \\ &= [\delta][P]\{ai\} \\ &= [B]\{ai\} \end{aligned} \quad (30)$$

Where $[B]$ is called strain matrix is a function of x and y co-ordinates.

The stress matrix can be expressed as follows:

$$\{AN\} = [D]\{e\} \quad (31)$$

The strain energy developed in the element is expressed by:

$$U_t = 1/2 * \iint [AN]^T \{e\} dx dy \quad (32)$$

Substituting the expression for $[AN]$ and e in the Equation(31) we get the strain energy.

$$\begin{aligned} U_t &= 1/2 * \{a_i\}^T \left[\iint [B]^T [D] [B] dx dy \right] \{a_i\} \\ &= 1/2 \{a_i\} [U] \{a_i\} \end{aligned} \quad (33)$$

$$\text{where } [U] = \iint [B]^T [D] [B] dx dy$$

Now substituting $\{a_i\}$ from Equation(28) into Equation(33) and finally making derivatives of strain energy U_t with respect to the nodal displacement parameters, we get the required element stiffness matrix $[S]$ and are given by:

$$[S] = [H^{-1}]^T [U] [H]^T$$

4.2.3 Overall Stiffness Matrix:

The element stiffness matrix relates quantities defined on the surface. Therefore, co-ordinate transformations are completely avoided and the overall stiffness matrix SFF of the slab structure is assembled by direct summation of the stiffness contributions from the individual elements. The degree of freedom for the overall stiffness matrix is obtained by substituting joint restraint form, the total number of displacement co-ordinates. The overall stiffness matrix is first partitioned so that the terms pertaining to the degrees of freedom are separated from those for the joint restraints. Then the matrix is rearranged by interchanging rows and columns in such a manner that stiffness corresponding to the degrees of freedom is listed first and those corresponding to joint restraints are listed second. Such a matrix is always symmetric. To computer time and storage, only the upper band of the stiffness matrix SFF (for free joint displacements) is constructed.

4.2.4 Load Matrix:

The vertical gravity load (mainly self-weight) is the major load for roof slab. The load intensity 'QL' is uniform over the area of a slab of uniform thickness. This load intensity 'QL' can be resolved into three components at a point in the three directions x, y and z as follows in a matrix. The above-distributed load is replaced by an equivalent nodal load matrix {AQ} for each element. This load matrix {AQ} is obtained by equating virtual work done by the uniform load {Q} and the nodal loads {AQ}. According to the standard formulae from texts:

$$\{AQ\} = \int_{-A/2}^{A/2} \int_{-B/2}^{B/2} [H^{-1}]^T [P]^T [Q] dx dy \quad (34)$$

Such a consistent load matrix will truly represent the distributed gravity load 'QL'. But the laborious process of Equation(34) can be avoided by using approximate overall nodal matrix {AQ}. This can be worked out as follows: The total vertical load on an element is assumed to be equally shared by its four nodes. The z components of this vertical load are the element nodal loads corresponding to displacements w. Con-

tributions from all elements connected at a node together form the final values of nodal loads for that node. Hence in the overall load matrix, out of five load values for each node, only the third will be non-zero.

4.2.5 Expression for Stresses / Moments:

From Equation(28) the expression for {ai} is found as follows:

$$\{ai\} = [H]^{-1} \{ui\}$$

Using these values of {ai} and combining Equation(30) with Equation(31), we get the matrix of resultant stresses / moments at any point (x, y) in terms of nodal displacements as follows:

$$\begin{aligned} \{AN\} &= [D] \{e\} \\ &= [D][B] \{ai\} \\ &= [D][B][H]^{-1} \{ui\} \end{aligned} \quad (35)$$

5. SOME PREVIOUS WORK ON FINITE ELEMENT METHOD

H.G.Kwak[20], worked on the finite element analysis of the monotonic behaviour of reinforced concrete beams, slabs and beam-column joint sub assemblages. H.M.Marzouk, ZhiweiChen[21] presented an analytical investigation on the behaviour of reinforced high-strength concrete slabs. A plasticity-based concrete model is used for the finite element analysis. M. W. Bari[19], worked on a slab element which is developed on the basis of conventional slab theory expressed in terms of rectangular co-ordinates and displacement. A computer program is developed for solution of finite element equations as well as to check rigid body modes and to obtain the results. M.M.Smadi, K.A.Belakhdar[22] developed a nonlinear finite element code to suite the analysis of normal and high strength concrete slabs. A software called NLFEAS (Non-Linear Finite Element Analysis of Slabs) was developed to predicate and study the three dimensional response of reinforced concrete slabs of different grades, variables and boundary conditions under monotonically increasing loads. Rifat Sezer[23], worked on finite el-

ements method (FEM) which was used for the non-linear analysis of reinforced concrete (R/C) plates under incremental loading up to failure load. Layered composite material model (LCMM) was used for the modeling of reinforced concrete plates. Piotr Rusinowski[24] worked on concrete slabs with openings which are usually designed with help of traditional rules of thumb proposed by building codes. Such methods, however, introduce limitations concerning size of openings and magnitude of applied loads. Furthermore, there is a lack of sufficient information and instructions are needed to design fibre strengthening of cut-outs in existing concrete slabs. Debajyoti Das [25], worked on a element formulation for the nonlinear analysis of thick rectangular and skewed reinforced concrete slab subjected to external mechanical load to predict its behaviour starting from the application of the load upto the failure. Priya Bansal[26], worked on strengthening of reinforced concrete (RC) structures is frequently required due to inadequate maintenance, excessive loading, change in use or in code of practice and exposure to adverse environmental conditions. Rifat Bulut[27], developed a finite element computer program to analyze slabs on elastic half space expansive as well as compressible soils. Mindlin orthotropic plate theory is adopted for structural analysis of ribbed or constant thickness slabs. The foundation soil is assumed to be an isotropic, homogeneous, and elastic half space. Gunjan Ashok Shetye[28], worked on response of reinforced concrete slabs subjected to blast loading as they can be used as protective structures around the main structure. Trevor D. Hrynyk[29], provided a procedure for improved nonlinear analysis of reinforced concrete (RC) slab and shell structures. The finite element program developed employs a layered thick-shell formulation which considers out-of-plane (through-thickness) shear forces, a feature which makes it notably different from most shell analysis programs. Khalil Belakhdar[30], worked on an implementation of a rational three-dimensional nonlinear finite element model for evaluating the behaviour of reinforced concrete slabs strengthened with shear bolts under transverse load. J. Ashley Warren[31], investigated on the tradeoffs between ac-

curacy and efficiency for various finite element modeling techniques used for determining mode shapes and natural frequencies of a fully composite, concrete slab on steel girder bridge. Ahmed Shaat[32], worked on the numerical simulation of the ultimate behaviour of 85 one-way and two-way spanning laterally restrained concrete slabs of variable thickness, span, reinforcement ratio, strength and boundary conditions. F.J Vecchio[33], investigated a nonlinear finite element shell analysis algorithms can be simplified to provide more cost effective approximate analysis of orthogonally-reinforced concrete flat plate structures. Shatha S. Kareem[34], worked on the numerical study to simulate the behavior six specimens of two-way RC slab of normal strength concrete (NSC), high strength concrete (HSC) and light weight concrete (LWC) with two steel ratio of 0.005 and 0.002 under concentrated load. SK Md Nizamud-Doula[35], worked on non linear finite element method using layered concept across the thickness has been adopted to study its suitability for the analysis of reinforced concrete slabs with special emphasis on skew slabs. Prakash Desayi[36], worked on two-way structural action, post-cracking non-linear behaviour and development of membrane action complicate the analysis and estimation of the ultimate strength of the slab. Neriman[37], worked on design of reinforced concrete structures against extreme loads, such as impact and blast loads, is increasingly gaining importance. Jianping jiang[38], developed the finite element model, which includes the bond slip, dowel action and the tension stiffening effects in reinforced and prestressed concrete slabs. Brian J. Henz[39], worked on the use of object-oriented programming techniques in development of parallel, finite element analysis software enhances software reuse and makes application development more efficient. B. Belletti[40], studied the new fib Model Code 2010 for the design shear resistance of a reinforced concrete (RC) structure can be evaluated through analytical and numerical calculation methods that fall into four different levels of approximations, the complexity and the accuracy of the calculated shear resistance increases with increasing the level of approximation. Mohamedien, A.R[41], developed a finite ele-

ment model for Double Skin Composite (DSC) panels subjected to quasi-static loading. A series of quasi-static finite elements models are used to analyze deformation and energy absorption capacity of such system, when perforated by rigid penetrator with conical nose shape. Songwut Hengprathane[42], worked on linear and nonlinear finite element analyses for the investigation of rectangular anchorage zones with the presence of a support reaction. The investigation is conducted based on four load configurations consisting of concentric, inclined concentric, eccentric, and inclined eccentric loads. Rodolfo Antonio Hutchinson Marin[43], created a Finite Element Routine for the Linear Analysis of Post-Tensioned beams using the program CALFEM developed at the division of Structural Mechanics in Lund University, Sweden. The program CALFEM and our own made files were written in MATLAB, an easy to learn and user-friendly computer language. Riyad Abdel-Karim[44], worked on SAP2000 program which is used to analyze single simply supported twoway ribbed slab models from 5 x 5m to 5 x 25m, supported on beams of different stiffnesses. This concentrates on the distribution of the loads to the perimeter beams and to the ribs in both directions. Y. M. Park[45], developed a equivalent frame method (EFM) for two-way slabs as a simple approximate method. Current design codes (ACI 318-05, Eurocode 2 and BS 8110) permit the EFM for the analysis of two-way slab systems under gravity loads, as well as lateral loads such as seismic loads. K.U. Muthua[46], provided the results of an analytical method proposed to predict the load deflection behaviour of partially restrained slab strips. The effect of deflection prior to yield line load on the development of compressive membrane forces was incorporated in the theoretical analysis. An experimental programme was designed to cast and test ten partially restrained slab strips with different edge rigidity. Freydoon Arbabi[47], provided a common procedure for the analysis of concrete buildings with two-way slabs is to reduce the slab along with other components of the building to a series of equivalent frames representing portions of the building between center-lines of spans. Young-Mi Park[48], studied on calibration factor for the effective beam width, which

is able to incorporate the effect of lateral stiffness provided by the edge beams of perimeter moment-resisting frames.

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